# The Binocular Summation Factor and its relevance for Deepsky Observing Part 1, Scientific Background 

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#### Abstract

Large-mirrored binoscopes are rare and amateur astronomers wonder about their benefits for deepsky observing. This often ends with one question only: how large are the two mirrors of a binoscope in comparison with a single, larger mirror? The theoretical answer to that question is partly dependent on the quantitative value of the so-called binocular summation factor. In part 1 of this article l'll address historic and scientific aspects of this factor as well as controversies about its value and interpretations. From the scientific literature regularly a value of 1.4 up to 1.7 emerges. Both the significance of the factor and its quantitative value are often misinterpreted by amateur astronomers. Sometimes a 1.41 high factor is interpreted as if one should multiply the aperture of a single mirror (or lens) with a factor 1.41 to predict the equivalent aperture of a two-mirrored binoscope (or binocular). However, another view maintains that one must multiply the aperture of one mirror with the substantial lower factor of $\sqrt{ } 1.41$ or 1.19 to obtain the two-mirror equivalent. This difference in interpretations is largely due to a complete lack of solid data. I have therefore directly compared a $2 \times 13$ inch binoscope with a 16 inch mono-mirrored telescope to address this question. In part 2 of the article l'll describe the results, as well as comparisons made by other observers. There, I will also discuss the relevance of the binocular summation factor in the context of other aspects of binocular vision.


## Introduction

How can one compare a large-mirrored binoscope with a 'mono-telescope' that has an even larger mirror? Binoscope owners may stress the increased signal to noise ratio you achieve by looking through a binoscope, resulting in an enhanced contrast of the images. This, however, appears not to provide a satisfactory answer. Instead, people, who have never looked through a binoscope, just want to know how one can calculate the equivalent aperture of the two binoscope mirrors. So, for example, if you have two 13 inch mirrors, adding up the surface areas equals a single, $\sim 18$ inch mirror. But what if you see only as much with this $2 \times 13$ inch binoscope as with a single 15 inch mirror? Why then bother and not simply buy for instance an 18 inch mirror with which you can see more than with 2 $x 13$ inch mirrors? This is a fair concern, since a lot of money or building effort is involved.

A seemingly logical assumption would be that 'one plus one equals two', meaning that you can see twice as much with two eyes than with one eye. A little reflection shows, however, that this cannot be true. When you close one eye during day light, it is obvious that you still see more than just $50 \%$ of before closing the one eye. Probably you see with one eye even as much as with two eyes, except for the lack of depth, which is caused by parallax. It is only when light is dim and objects are visible at threshold levels that one start to note a difference between viewing with one or two eyes. This phenomenon has been the subject of a long history of vision research. This scientific research is in particular medical oriented. Naturally the question is of importance in cases of (partial) blindness to one eye, caused by accident or disease. Beside these practical, medical aspects, there is also a long standing interest into the theoretical aspects of 'binocular' vision versus 'monocular' vision. Part of this research revolves about the question of how one could quantify the differences of viewing with one or two eyes. In a lot of quantitative models the binocular summation factor and its quantitative value plays an important role.

## Binocular summation factor

Binocular summation is the process by which the brain combines the information that they get through incoming signals in the left and right eye. By means of binocular summation the threshold value for the detection of faint objects is lower with two eyes than with one eye. Statistically there is an advantage for the detection of a weak signal when two detectors are used instead of one detector. This
advantage is $\sqrt{ } 2$, or 1.4 , called the binocular summation factor. On Bruce Sayre's website an excellent lecture given by dr. Thomas Salmon is quoted. In this lecture the theory is summarized.

Early experiments tried to pinpoint the advantage of binocular vision quantitatively (Pirenne, 1943). It was shown that with binocular detection a faint light signal is 1.4 times better observed than with monocular detection. The theory that was based on these early experiments is called the probability summation theory. To give an idea how such experiments were performed and interpreted, 'lll summarize some of the findings. Pirenne used flashes of light with different brightness, for the duration of a few milliseconds. The observed frequency of seeing the flash of light were noted for the left eye only, for the right eye only and for both eyes. Here is one telling example (Pirenne, 1943):

## Observed frequency of seeing

Left eye $\quad 25 / 125=0.198$
Right eye $\quad 71 / 275=0.258$
Both eyes $\quad 62 / 164=0.378$ (this is a factor 1.66 better than the average of left/ right frequencies)
Now assuming that the probabilities of seeing signal with the left eye $\left(P_{\mathrm{I}}\right)$ or the right eye $\left(\mathrm{P}_{\mathrm{r}}\right)$, are independent, one can predict the probability of seeing with both eyes ( $\mathrm{P}_{\mathrm{b}}$ ). The definition of probability of detection this signal with both eyes is: $P_{b}=P_{r}+P_{l}-\left(P_{r} \times P_{1}\right)$. If the above observation for the left and right eyes are calculated, $\mathrm{P}_{\mathrm{b}}=0.198+0.258-(0.198 \times 0.258)=0.405$. This is pretty close to the observed 0.378 . When the above and other experimental data are plotted, the figure below emerges. The $B$ line is calculated from the observation values for the left $(L)$ and right $(R)$ eyes. At log brightness 1.0 , the above explained example is plotted.

There are a couple of things to note. The observed frequencies with both eyes closely fit the predicted probabilities that were calculated with the formula above. Hence it was concluded that the increased probability of seeing with both eyes (the open circles in Fig. 1) can be explained with a statistic summation only and that no other, physiological fusion mechanism in the brain has to be responsible. Or, in other words, the two eyes are just seen as independent detectors. Here the term probability summation theory stems from.

A second point I want to stress is that the curves for the left and right eye differ significantly. Although this can be expected for two different eyes with different sensitivities etc., this point will come back later.


Figure 1. Pirenne's experimental data that fit the predicted values for the increased probability of seeing a light flash with two eyes (B), as compared with the observed frequency of seeing with the left (L) or right (R) eye only. The circles through which the B line runs, are the calculated Pb values.

Finally, the increased probability of seeing with both eyes depends highly on the brightness of the stimulus. In Salmon's notes, he arbitrarily used a probability of 0.6 of seeing with each eye. That means that the total probability for two eyes is $0.6+0.6-(0.6 \times 0.6)=0.84$. That is 1.4 times more than 0.6 , so here we have the much quoted $\sqrt{ } 2$ or 1.4 large binocular summation factor. However, the probability of detection could be anything, depending on what the person is looking at. If it is very difficult to see an object, the probability of detection would be very low, for example 0.1 . If the object is easily visible, the probability would be nearly 1.0. In the introduction I referred in this context to binocular vision during daylight. If you look during daylight at a bright object, it won't matter much whether you look at it with one or two eyes. So the probability will be 1.0. That means that a total probability for seeing with both eyes is 1.0 too (namely $1.0+1.0-(1.0 \times 1.0)=1.0)$. So it is only at increasingly dimmer lights that the probability for seeing with both eyes will become higher. For instance, when the probability for each eye is 0.3 , the probability for seeing with both eyes is $0.3+0.3$ $-(0.3 \times 0.3)=0.51$. This is a factor 1.7 larger than the probability of seeing with one eye only. These trends are also visible in figure 1.

Generally, when vision scientist conduct experiments to determine the threshold for detection, they often use a value of 0.5 . Why? Since there is not a clear intensity level that you can call a "threshold," they arbitrarily define the threshold as the intensity at which you get a $50 \%$ probability of detection or 0.5 . The probability that two eyes detect the signal is now $0.5+0.5-(0.5 \times 0.5)=0.75$. And now we have a 1.5 factor increase as compared to the 0.5 probability of seeing with one eye only.

Also the much quoted 1.4 binocular summation factor is in fact largely dependent on how dim the observed objects are and is often used out of convenience. This is because this value coincides exactly with the increase in aperture diameter of two combined mirror surfaces. Take for instance my two mirrors with a diameter of 13 inch each. The surface of each mirror is or $\pi(1 / 2 \mathrm{~d})^{2}$, or $\sim 132.6 \mathrm{inch}^{2}$. The combined surface of the two mirrors is thus $\sim 265.3$ inch $^{2}$. And this is equivalent to one mirror with a diameter of $\sim 18.4$ inch. And that is exactly a factor $\sqrt{ } 2$ or 1.41 larger than my 13 inch mirror diameter.

## Binocular summation, binocular facilitation and other aspects of binocular vision

It is known that many visual cortical neurones are binocularly connected in higher primates. Since there exist functional and physical interactions between visual neurones from the two eyes it is hard to believe that statistics and probability are the only explanation for binocular summation. So while the probability summation theory is still perceived in the scientific literature as a valid approach, there are additional factors that influence the value of the binocular summation factor. For instance, there are conditions, in which the increase in binocular sensitivity is greater than could be explained by probability summation alone. Optimal summation occurs when 1) corresponding points on the two retinas are stimulated with like targets or stimuli, and 2) when the stimuli are presented to the two eye simultaneously, or at least within $\sim 100 \mathrm{msec}$ of each other. In these cases the activity of the brain is enhanced more than the sum of both brain activities that are provoked by each one eye separately. If there is any advantage above the mentioned binocular summation factor of 1.4 , this is attributed to this mechanism, which is called binocular facilitation or neural summation.

Furthermore, Campbell and Green (1965) provided another explanation of why binocular summation should lower the visual threshold by a factor of 1.4. They argue that by combining the input from two eyes, neural signals would be added while background neural noise (assumed to be random and uncorrelated) should partially cancel. They predicted and measured that this process alone would cause binocular thresholds to be lower by a factor of $\sqrt{ } 2$ or 1.4 (Figure 2).

Therefore, the often recurring 1.4-fold improvement in visual function could be explained by either probability summation, an increase in signal-to-noise ratio or neural summation. Any improvement by more than this 1.4 fold would indicate that neural summation or some other form of physiological summation is involved.


Figure 2. Campbell and Green (1965) contrast sensitivity tests. The open circles at the top represent the binocular/ (mean) monocular sensitivities. The straight horizontal line is the average of these ratios at different spatial frequencies, which is $\sqrt{ } 2$ ! (In fact $1.418 \pm 0.021$ for three experiments with two different people).

Since these landmark studies, a lot of scientific research has been conducted concerning binocular versus monocular vision. At least five different models have been proposed (Meese et al, 2006) to explain binocular summation. Sometimes experiments are conducted that show that binocular summation exceeds the factor $\sqrt{ } 2$ (Meese et al, 2006). In fact, factors of 1.7 and up to 2 been reported (see Frisen and Lindblom, 1988). But as far as I can tell, no simple interpretation has come forward as yet. There are a number of reasons for that.

1. Methodologies range widely, signifying that it is not an easy task to design experiments that unequivocally address and resolve the issue properly. One experiment may have let subject dark adapt for 30 minutes, but another may have dark adapted for only 10 minutes. In one experiment, the dim light may have been presented for just 1 second, but in another, for 5 seconds. All these variables can affect results and the protocols used are far from standardized. In my case of estimating limiting magnitudes (see below) it is of importance that my eyes have to be well darkadapted in order to detect the weakest star possible. This usually takes at least 15 to 30 minutes.
2. Aspects of vision such as the detection of threshold levels of light, contrast or resolution benefit to a different extent by binocular vision. A point source would be perceived only by a small area of the retina, but a larger source would include areas of the retina that might respond differently. For example, the physiology of the central $1^{\circ}$ of vision is very different from that a few degrees peripherally. This makes 'fixing' one single binocular summation factor for all these aspects also a difficult affair.
3. Our two eyes are rarely identical, this can already be seen in the 1943 Pirenne data. The 1.41 factor may only be valid when both eyes are equally sensitive and optimal. But it will in the extreme decrease to 1 when one eye has no sensitivity at all (Nelson-Quigg et al., 2000).
4. In a number of studies large differences are reported between individuals. The spread is so large that one study concludes that there may be not a single binocular summation constant at all (Frisen and Lindblom, 1988). They also concluded that the degree of binocular summation is related to the complexity of the visual task. For instance, they found that the binocular summation
factor was significantly smaller in resolution tests than in detection tests. Similarly, when an even more complex parameter such as contrast is taken as read-out, the resulting binocular summation factor is higher than the often quoted 1.4 (Meese et al, 2006).
5. It is telling that some scientific papers use more accurately phrases such as "the $\sqrt{2}$ ratio of binocular to monocular contrast sensitivity" (Anderson and Movskon, 1989). Or "the binocular summation (the ratio of binocular to monocular contrast sensitivities at threshold) is $\sim 1.7$ " (Meese et al., 2006).

## Interpretations of the binocular summation factor by amateur astronomers

How has the $\sqrt{ } 2$ or higher value for the binocular summation factor been perceived by the amateur astronomy community? In most instances the reasoning is simple: just add the two areas of the mirrors. But others say no, this is not the way things work. For instance Ed Zarenski on Cloudy Nights (Zarenski, 2006) states this: "these factors are applied on the aperture delivering light to each eye, not the total area of the two apertures delivering light to both eyes. What I mean is this; you would not add the area of two 70 mm lenses to get $4900+4900=9800$, then take the $\sqrt{ } 9800$ to find total light delivered from a total 99 mm aperture. The light is delivered from a 70 mm aperture to each eye. The binocular summation factors are applied to that 70 mm aperture".

He thus implies that two 70 mm lenses are not equivalent to $\sqrt{ }\left(\mathbf{A}^{2} \times \mathbf{2}\right)=\sqrt{ }\left(2 \times 70^{2}\right)=\sqrt{ }(2 \times 4900)=99$ mm . Instead the area of 4900 times is multiplied with the summation factor 1.41 , which predicts the 'novel' combined area for two eyes. This results in an aperture of $\sqrt{ }(4900 \times 1.41)=83.2 \mathrm{~mm}$, which is $\sqrt{ } 2$ times the aperture of 70 mm . In formula this becomes $\sqrt{ }\left(A^{2} \times 1.41\right)$. So instead of multiplying the diameter of the mirror by $\sqrt{ } 2$, the diameter is multiplied with a factor of $\sqrt{ } 1.41$ or 1.19 . This would imply that my $2 \times 13$ inch mirrors are equivalent to a $\sim 15.5$ inch mono telescope instead of $\sim 18.3$ inch.

This interpretation is restricted to an amateur astronomy audience that visit internet forums such as Cloudy Nights. However, it has been taken to a wider audience by Phil Harrington in his book Cosmic Challenges (Harrington, 2011). He takes over Zarenski's interpretation and uses the formula $\sqrt{ }\left(A^{2}\right.$ $x 1.41$ ). He uses this formula to calculate the equivalent telescope aperture of a number of binoculars and simply multiplies the aperture of one lens with 1.19.

Without choosing between which one of these views is correct, there are a couple of things to note:

1) As argued, there no such thing as a singular binocular summation factor. Its quantitative value depends for instance largely on which parameters are used as read out, such as resolution, detection etc. The 1.41 factor is the result of often convenient assumptions. Up to values of 2 have been reported. And a factor 2 would imply a 1.41 times larger mono-mirror aperture.
2) Probably of more importance is the question whether one can translate the binocular summation factor in a rather linear fashion to predict a comparable mono-mirror aperture. Vision scientists will never embark on such an endeavour and with good reason. Grossly simplified, they are concerned with is 'how much more or better you see with two eyes than with one eye'. From their perspective it makes no sense to translate that question into 'how big should one eye be to replace or be equivalent to two eyes'. After all, we do have only two eyes and not one larger 'cyclops' eye to compare with, so why bother with this question. In short, taking any value of the binocular summation factor and use it to predict how big a comparable mono-mirror would be, is an INTERPRETATION of the factor's meaning. It stems from an understandable obsession of amateur astronomers who want to know whether it is literally worth it to go through the trouble of buying/ making a binoscope.

Given these considerations I started to doubt the rather one-dimensional approach of 'there is a summation factor of 1.41 and that translates to a 1.19 increase in aperture'. In particular since I had
the impression that I saw more with my $2 \times 13$ inch binoscope than I could expect from a 1,19 times large, 15.5 inch mono-mirror. Hence I started to think of how I could approach this comparing a binoscope directly with a larger telescope harbouring a larger, single mirror.

## Conclusions

Scientific research into the relevance and the value of the binocular summation factor has a long history. It is clear from the extensive literature that it is very hard to assign an exact and single value to this factor. This may not be surprising given that binocular vision involves many different aspects. Not only the physical components of the eyes, but also complex neural processes play key roles in binocular vision. Therefore, one single binocular summation factor is probably not sufficient to cover all aspects of binocular vision. Furthermore, there is an amazing range in how different individuals perceive aspects of binocular vision. Therefore utmost care must be taken with generalizing conclusions. In particular sweeping statements and simple formulae to compare binoculars (or two binoscope mirrors for that matter!) to one larger mirror are almost certainly misleading at best. This, however, appears to be the unsatisfactory status in amateur astronomy. One way out of this could be to directly compare a large binoscope with a large mono-telescope. At least for point light sources, i.e. stars, one can determine the limiting magnitudes and from that calculate the binocular summation factor for that specific situation. In part 2 of the article l'll present such direct measurements. In part 2 l'll also address the relevance of the binocular summation factor for deepsky observing, in comparison to other factors that favour binocular viewing.

## Acknowledgements

A number of people commented on the questions that I sent them. Those questions concerned the binocular summation factor and its relevance for visual observations of deepsky objects. I want to thank them for their time to write me back and share their views and insights with me. In alphabetic order those are Phil Harrington, Bruce Sayre, Gary Seronik and Mark Suchting. I am particularly indebted to Mel Bartels, Jan van Gastel and Dr. Thomas Salmon. They were very supportive and encouraged me to continue searching the literature and their comments eventually shaped the contents of this article. Mel Bartels also suggested to directly measure the limiting magnitudes and allowed me to cite his results.

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# The Binocular Summation Factor and its relevance for Deepsky Observing Part 2, Measurements and Observations 


#### Abstract

Large-mirrored binoscopes are rare and amateur astronomers wonder about their benefits for deepsky observing. This often ends with one question: how large are the two mirrors of a binoscope in comparison with a single and larger mirror? The theoretical answer to that question is partly dependent on the quantitative value of the so-called binocular summation factor. In part 1 of this article I addressed historic and scientific aspects of this factor as well as controversies about its quantitative value and significance. The controversy is largely due to a complete lack of solid data. I have therefore directly compared a $2 \times 13$ inch binoscope with a 16 inch mono-mirrored telescope in order to determine their respective limiting magnitudes. Here, in part 2 of the article I describe the results. A popular view amongst amateur astronomers predicts that $2 \times 13$ inch mirrors should be equivalent to a single 15.5 inch mirror. However, I found that limiting magnitudes of the $2 \times 13$ inch binoscope are consistently higher than of a single 16 inch mirrored telescope. Also, on extended objects, such as galaxies, the differences are even larger, in favour of the binoscope. I discuss what these findings imply for the binocular summation factor. I also discuss the binocular summation factor in the context of other aspects of binocular vision and argue that the factor itself is of only limited relevance for deepsky observing.


## Introduction

How can one compare a large-mirrored binoscope with a 'mono-telescope' that has a larger mirror? As explained in Part 1 of the article, the binocular summation factor and its quantitative value may play an important role in how to compare the two mirrors to one larger mirror. The scientific literature is very careful in assigning a single value to the binocular summation factor and its interpretation. In the amateur astronomy community a prevailing view is put forward by Zarenski (2006) on Cloudy Nights and Phil Harrington in his book Cosmic Challenges (Harrington, 2011). They assign a simple formula to the comparison between two mirrors compared to one larger one. This formula is $\sqrt{ }\left(A^{2} \times 1.41\right)$. Here, A stands for the aperture of one binocular lens (or binoscope mirror). This, in short, says that one must multiply the diameter of one lens/ mirror with a factor 1.19 to obtain the diameter of a comparable single lens/mirror. However, observers who have directly compared large binoscopes with comparable mirrors claim that their impression is that this factor is too low.

One way to address this controversy is to directly compare a large binoscope with a large monotelescope. At least for point light sources, i.e. stars, one can determine the limiting magnitudes and from that calculate the binocular summation factor for that specific situation. Here I present such direct measurements. Finally, I also address the relevance of the binocular summation factor for deepsky observing, in comparison to other factors that favour binocular viewing.

## Measuring the limiting magnitudes of a binoscope and a comparable single mirror telescope

So the question is: how does the size of two binoscope mirrors compare to one larger mirror. One way to test this is to determine limiting magnitudes of stars under equal observation conditions and compare these for both a binoscope and a comparably larger single mirror telescope. To determine the limiting magnitude one simply determines the faintest star one can still see with either the binoscope or the comparable one, larger mirror. The resulting differences in limiting magnitude are thereby an indirect measure for the value of the binocular summation factor. In a mail exchange with Mel Bartels, he proposed that he would directly compare a binoscope and an equivalent monomirrored Dobsonian telescope to determine the limiting magnitudes of either instrument. I followed up his suggestion and compared the $2 \times 13$ inch binoscope with a 16 inch mono-Dobsonian telescope for limiting magnitudes (see Figure 1). The mirrors of the $2 \times 13$ inch binoscope are f/5.0, giving a focal length of 1650 mm . The 16 inch mirror is $\mathrm{f} / 4.5$, but I use a Paracorr coma corrector, which transforms
the mirror into a $f / 5.2$, and an effective focal length of 2080 mm . When using the same eyepieces, the respective focal ratios of $f / 5$ and $f / 5.2$ provide exit pupils that resemble each other closely. So, for example, when using 10 mm Ethos eyepieces, the exit pupil with the binoscope is $10 / 5=2$, while the exit pupil with the 16 inch $10 / 5.2=1.92$. These exit pupils are almost the same. This is important, since when viewing point light sources such as stars, the exit pupil determines the degree of background 'blackness'. By coincidence the background darkness in each telescope is thus more or less the same, allowing a proper comparison. Only the magnification in each telescope will differ because of the different focal length.

I determined the limiting magnitude for each telescope during two nights. The first night had almost perfect conditions, with an SQM of approaching 22.0 (a naked limiting magnitude of 7.0) and a very high level of transparency. I chose a star field close to Polaris. The second night conditions were somewhat less, with a SQM of 21.5 (a naked eye limiting magnitude of 6.6). Transparency was good. I chose two star field, surrounding NGC 7448 and NGC 7678 in Pegasus.


Figure 1. At the left my 16 inch $f / 4.5$ mono-Dobsonian telescope and at the right the $2 \times 13$ inch $f / 5.0$ binoscope. For more pictures of these instruments see my website (http://arieotte-binoscopes.nl/Binoscopes.htm)

The results are shown in Table 1. The limiting magnitudes were determined in the respective star fields with the $2 \times 13$ inch binoscope (column $2 \times 13$ ) and the 16 inch mono-mirrored Dobsonian telescope (column 16). As can be expected, the limiting magnitudes from the second session are somewhat lower than during the first session, due to the lower sky blackness. Also, with a smaller exit pupil (higher blackness of the background sky) the differences appear to become somewhat larger. The main conclusion is though that, when taken as a whole, the limiting magnitudes of the binoscope are consistently higher than that of the single mirrored 16 inch telescope. This is not compatible with the prevailing view that the $2 \times 13$ inch binoscope should behave as one $13 \times 1,19=15.5$ inch telescope.

|  |  |  |  |  | Observed limiting magnitudes |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | Aperture (inch) |  |  |  |  |
| Date | SQM | Eyepieces | Exit pupil ${ }^{1}$ | Magnification ${ }^{2}$ | 16 | $2 \times 13$ | $1 \times 13^{3}$ | $\Delta$ factor with 13 inch $^{4}$ |  |
| 1-5-2013 | 22.0 | 27 mm Panoptic | 5.4 | 61-77 | 15.5 | 15.8 | 15,1 | 1,39 |  |
|  |  | 10 mm Ethos | 2.0 | 165-207 | 15.9 | 16.4 | 15,5 | 1,53 |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
| 1-8-2013 | 21.5 | 27 mm Panoptics | 5.4 | 61-77 | 14.4 | 14.9 | 14.0 | 1.51 |  |
|  |  | 10 mm Ethos | 2.0 | 165-207 | 15.5 | 16.0 | 15.1 | 1.51 |  |
|  |  | 6 mm Ethos | 1.2 | 275-345 | 15.8 | 16.4 | 15.4 | 1.58 |  |

1. Calculated for the $f / 5.0$ binoscope. With a Paracorr the 16 inch $f / 4.5$ telescope has a $f / 5.2$ ratio, the exit pupils are corrected for that.
2. The first magnification is for the $2 \times 13$ inch, the second for the 16 inch $f / 4.5$, with Paracorr
3. To calculate the limiting magnitude for a single 13 inch mirror, in comparison to the 16 inch mirror, the following formula is used: $\mathrm{M}-5^{*} \log$ (aperture 400/aperture330). For instance for 10 mm Ethos at 1-5-2013: $15.9-5^{*} \log (400 / 330)=15.9-5^{*} 0.0834=15.9-0.417=15.48$ (15.5 is written down!)
4. To calculate the increase of the two 13 inch mirrors as compared to the one 13 inch mirror, the following formula is used: $10^{\wedge}\left(1 / 5^{\star}\left(M \_\right.\right.$bino-M_mono) ). For instance for 10 mm Ethos at 1-5-2013: $10^{\wedge} 1 / 5^{*}(16.4-15.48)=10^{\wedge} 0.184=1.53$.

Table 1. Results that show the limiting magnitudes when using either the $2 \times 13$ inch binoscope or the 16 inch mono-mirrored Dobsonian telescope. See the text for explanations.

## Observations by others

During the same period Mel Bartels and some experienced observers compared a $2 \times 8$ inch binoscope with either a 12 or 13 inch mono-mirrored Dobsonian telescope. The difference in equivalent apertures between the $2 \times 8$ inch and the 12 or 13 inch mono-mirror is a factor 1.5 and 1.62 respectively. They concluded that in terms of limiting magnitudes the binoscope was just a tad less' than the larger mono-mirrored telescopes (Mel Bartels, personal communication). Combining their data with mine, it appears safe to state that an increase in 1.4 to 1.5 in aperture does more to justify the observations than a 1.19 increase, which would result in a predicted 9.5 inch aperture mirror.

While the above is concerned with point light sources only, a different picture emerges with extended objects, such as nebulae and galaxies. The problem is that differences on extended objects are much harder to quantify accurately. However, Mel Bartels continues to say that "but the nebulosity was equal or better in raw detail and *every single observer* there agreed that aesthetically the 8" binos were better on extended objects" (Mel Bartels, personal communication). When a quantitative value is attached to the increase in equivalent aperture when extended objects are concerned, a factor of 1.7 was more often cited.

During the 2014 Oregon Star Party Telescope Walkabout, Jerry Oltion demonstrated his $2 \times 12.5$ inch during three nights (http://www.bbastrodesigns.com/osp14/osp14.html\#Jerry Oltion). "The binocular effect proved striking, gaining a magnitude, obvious on all objects and stars, none more so than the galaxy cluster Hickson 84 where the bino view showed galaxies fainter than 17th magnitude whereas a single mirror struggled to reach 16th magnitude". (that is an increase $10^{\wedge} 1 / 5^{*}(17-16)=10^{\wedge} 0.2=$ 1.58 (my calculation)). Observers judged that "the scope performed equal to that of a 18 inch to 24 inch scope, depending on object". That's a factor 1.44 to 1.9 difference in aperture! This is in line with what I describe in this article. Furthermore, "the Dumbbell Nebula was quite striking - one of the best objects in the scope". Or, "one object that stood out better in Jerry's binoscope than any other scope regardless of size was M31, the Andromeda Galaxy. The dark dust lanes were very striking".

Also Peter Vercauteren states that: After having used my 18" bino-Dobson for over a year now and having compared it extensively with other instruments, including a 27" mono-Dob, I can confirm that a $18^{\prime \prime}$ bino is at least equal, if not superior to a $25^{\prime \prime}$ mono. On faint objects I was even able to discern more detail than in the 27" (https://www.cloudynights.com/topic/588506-binoquadroculars/page-2).

I myself can confirm that extended objects benefit the most from binocular viewing. For instance, with a 20 inch telescope I probably saw M51 as slightly brighter. But with the $2 \times 13$ inch binoscope I see much more details and contrast within the arms and halo of M51 than with the 20 inch. This observation does not stand on its own. Similarly, I saw the very faint arms of M81 much more pronounced and extended with the binoscope than with the 20 inch. And even more dramatic are views of M31, the Andromeda nebula. The extremely large and faint halo is readily visible in both the 2 $x 13$ inch binoscope and in the 20 inch. But, with the binoscope a sharp transition between the edge of the halo and the space beyond is visible. In other words, with the binoscope one sees better where the M31 halo ends. This is easily observed when one scans at low speed through the halo. All these observations are the result of the highly increased contrast that can be gained with a binoscope. Below in Figure 2 I give an impression of how I observed the M27 Dumbbell nebula through either a 20 inch $f / 4.0$ mono-telescope and the $2 \times 13$ inch binoscope. What stands out is not so much an overall brightness, but the highly increased contrast in details such as filaments.


Figure 2. At the left a drawing of the M27 Dumbbell nebula with a 20 inch $f / 4$ mono-Dobsonian telescope and at the right a drawing the Dumbbell nebula seen through the $2 \times 13$ inch $f / 5.0$ binoscope.

Although mostly qualitative in nature, all these observations point into the same direction. On extended objects, two mirrors in a binoscope are at least comparable with a 1.4 to 1.5 larger aperture of a mono-mirror. This is significantly better than the predicted 1.19 increase in aperture.

What do these observations imply for the binocular summation factor and its use?
So where does that leave us in terms of the binocular summation factor and its significance? In the prevailing interpretation, the aperture of one mirror should be magnified by a factor 1.19. This stems from the formula $\sqrt{ }\left(A^{2} \times 1.41\right)=A \times 1.19$ in which 1.41 would be the binocular summation factor. Above cited observations, however, claim increases in aperture of factors ranging between 1.4 and 1.7. With this specific use of this formula it would imply a binocular summation factor of between 2 and 2.9. As explained in the first article, the 1.41 value for the summation factor is rather arbitrary and values between 1.1 and 2 have been reported. Nonetheless, a value of 2.9 seems to be excessive and unlikely.

It seems more likely that the rather simplistic use of taking a binocular summation factor and use it to predict a larger, comparable mono mirror is not valid. As pointed out in the first article, all vision
research concerning the binocular summation factor involves the comparison between a left and right eye and the combination of these two eyes. The left and right eye may be more or less comparable or one of them may be impaired, but never are the two eyes being compared to a single BIGGER eye. This is impossible and so no scientific research exists that approaches the question from this angle. Still, this is what amateur astronomers try to achieve. So unless all observations that are reported above are invalid, the $\sqrt{ }\left(A^{2} \times 1.41\right)$ formula cannot be correct. It also leaves the question of the quantitative value of the binocular summation factor in this specific context wide open.

## The binocular summation factor in comparison to other factors that favour binocular viewing

Assuming that the binocular summation factor would range up to a rather high value of 2 , what is its real impact for deepsky observing? This value would in the often cited formula result to an approximately equivalent to a doubling of the mirror aperture. So, the $2 \times 13$ inch binoscope would be equivalent to a 18.2 inch mono-mirror. As shown in Figure 4, this doubling in light-gathering area of a mirror results on average in an increased limiting magnitude of $\sim 0.7$ (Bartels, 2012). To put this number into perspective: when I am in my hometown I achieve a deplorable limiting magnitude of 4.7, if I'm lucky. In rural France a limiting magnitude of 6.7 is easily within reach. This gain in 2 full magnitudes dwarfs the at most 0.7 increase in limiting magnitude that can be achieved by changing from an equivalent mirrored mono telescope to a binoscope. Or, if the binocular summation factor in terms of light gathering capacity is the only argument to go for a binoscope, my advice would be:
'don't bother'.


Figure 3. Limiting magnitudes that correspond with different aperture mirrors. In yellow the apertures in inches diameter are given, indicated by the upper row of numbers. In bleu are double apertures (double surface areas), calculated in aperture diameters, indicated by the lower row of numbers. Note that the difference between the yellow and blue bars I consistently ~0.7 magnitudes. Also an increase from 12 inch to 16 inch (approximately a doubling in aperture) results in an 0.7 increase in limiting magnitude. The numbers are derived from Mel Bartels (Bartels, 2012).

What then would be the principal factors that determine the benefits of a binoscope over a largemirrored mono-telescope?

1. The most important factor is the better signal to noise ratio that leads to the perception of a darker sky background. This is the single most important factor that favours a binoscope in comparison to a large mirror single telescope. This result translates as a difference in the contrast gain of a binoscope versus a large single mirror telescope. What does this mean? A stimulus that is not originating from an astronomical object (for instance 'light noise' from light pollution) could be interpreted by one eye as a bona fide signal. But the chance that such random noise signals hit two eyes simultaneously and reach the brains is very small indeed. In other words, when looking with two eyes, the brains do have to suppress much less background noise created by light pollution. And this automatically translates itself into a darker sky background, even in my lightpolluted hometown. The resulting improved contrast is in particular relevant for the observation of very faint galaxy halo's or arms. Within these faint, extended objects many more details become visible and the contrast within the objects is greatly enhanced. Neural summation is probably responsible for this phenomenon. Here a binoscope has also an distinctive advantage above a binoviewer that simply splits ONE light signal into two. With a binoviewer the reduction in the random noise signal principally cannot take place!
2. Stereopsis is the ability see depth. Because of the different positions of the eyes, an object is viewed by each eye from a slightly different angle (parallax). This creates a spatial 3D effect. How closer the object is, the larger the angle and how greater the 3D effect. But unfortunately, astronomical objects are so distant that there is no such thing as parallax there and consequently no 'real' stereopsis. But, there is a related phenomenon called chromatic stereopsis or chromostereopsis. This is caused by the slightly different breaking in the eye lens of for instance red versus blue light. As a consequence red and blue light focus in a slightly different place on the retina. This effect is different for each eye and it therefore appears as if red stars stand a bit closer than blue stars. When looking through a binoscope, chromostereopsis hereby creates an illusion of depth, although this is completely artificial.
3. The principally wider field of view that can be achieved with a binoscope can hardly be achieved with a large mono-telescope, a point which is often stressed by Mel Bartels. Indeed, looking through a binoscope, this is a beautiful effect that is immediately obvious. But why is that? If you take the example of my $2 \times 13$ inch binoscope, with $f / 5.0$ mirrors, the use of two 10 mm Televue Ethosses delivers a $165 \times$ magnification and a 0.61 degree true field. Suppose this binoscope has an approximate equivalent of a 18 inch mono-telescope, also being f/5.0. Now the 10 mm Ethos would deliver a $225 \times$ magnification and a 0.45 degree true field, which is only half of the true field you see with the binoscope. To achieve a $165 \times$ magnification and a 0.61 degree true field, the 18 " needs to be f/3.7. And since this creates massive coma, a coma-reducing Paracorr will be needed. As a consequence, the mirror needs to be even $f / 3.2$ for these same magnification and true field (with thanks to the Televue Eyepiece Calculator!). Now consider what will be the costs of such a steep mirror!

There is another aspect here that is rarely addressed. In humans, the horizontal binocular visual field is 120 degrees. But there is an additional 45 degrees monocular field on each side of the binocular field. So, there is a total of a 90 degrees field that is non-binocular, but that is seen by looking with two eyes. By looking with one eye only, this is just a mere 45 degrees of the one eye. And even though you cannot encompass the entire 120 plus 90 degrees at one glance, you will observe it peripherally and it adds to the feeling of being present in the picture. Of course, the choice of eyepieces is important here: with two 50 degree apparent field of view eyepieces, you will just see the field stops. But the effects are very obvious when two Ethos eyepieces with a 100 degree apparent field of view are used.
4. The sheer comfort of observing with two eyes. There is also the 'ordinary' effect of the increased comfort by looking with two eyes instead of with one eye, and another squeezed eye. Sustained
and concentrated viewing at faint details with two eyes, without the strain of looking with one eye only, is more pleasant and more relaxing.
5. The gain in limiting magnitude, or a gain in aperture, which makes a binoscope comparable with a larger mono-telescope. This is the topic of this article. Unfortunately, as explained, this aspect gets most of the attention.

## Conclusions

The use of a specific value of the binocular summation factor to predict how the aperture of two binoscope mirrors compare with a single, larger mirror is misleading at best. First of all, experimentally determined values of the binocular summation factor range considerably and this influences theoretical predictions. Secondly, scientific experiments that investigate vision phenomena such as the binocular summation factor are based on the use of a single left or right eye versus the use of two eyes. And they are not used to predict how big one single, larger 'cyclops'eye would be as compared to our two eyes. This is, however, precisely what amateur astronomers attempt and it leads to rather odd results.

Direct comparisons between large binoscopes and larger, single mirrored telescopes indicate that the increase in the aperture of the larger, comparable mirror ranges between a factor 1.4 and 1.7. This is at odds with the commonly used $\sqrt{ }\left(A^{2} \times 1.41\right)$ formula that predicts a $1.19 \times$ gain in aperture of the larger mirror. Either these observations are wrong or the use of this formula and its predictions. I tend to believe the last.

I further argue that, whatever its quantitative value, the binocular summation factor has only limited relevance for deepsky observing. The increased signal to noise ratio that is achieved with a binoscope is probably more important for the beneficial effects of looking with two eyes through a binoscope. Still, the claim of a meagre 1.19 improvement of a binoscope (or binocular) in terms of light-gathering capacity probably puts of many potential users/ builders of binoscopes. This is regrettable, to say the least.

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